

A review of heavy-heavy spectroscopy

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Calculations of the heavy-heavy spectrum present a good opportunity for precision tests of QCD using lattice techniques. All methods make use of a non-relativistic expansion of the action and its systematic improvement to remove lattice artefacts. There was convincing demonstration this year that these methods work and that the associated perturbation theory is well-behaved. Comparison to experimental results yields an accurate value for the lattice spacing, a , a key result in the determination of α_s , and (for the first time this year) the mass of the b quark (4.7(1) GeV).

1. INTRODUCTION

The spectrum of heavyonium states is an ideal place to provide a precision test of QCD from first principles. Very accurate calculations are possible using lattice techniques. The reason is that *both* statistical and systematic errors can be made small, something we are nowhere near achieving for light hadron calculations.

The key ingredients are

- A non-relativistic formulation appropriate to the physics.
- Moderately-sized lattices at moderate values of β with perturbatively improved actions.
- High statistics from $\mathcal{O}(100)$ configurations and the use of multi-exponential fits to multiple correlation functions.

Results this year demonstrate clearly that perturbatively improved actions work and give continuum results when values of the lattice spacing are not very small. This is particularly useful for heavy quark systems where lattice discretisation errors are relatively large. For light hadron systems, however, its use would allow calculations at lower values of β and smaller volumes than are currently employed, which might be an advantage. Another clear result is the greatly increased confidence in the ground state that is obtained from a multiple exponential fit to multiple correlation functions. Such a fit is not often possible

at the level of statistics employed in current light hadron calculations.

Results were presented by four groups this year. The groups differ in the action employed for the heavy quarks, the level of improvement of the action used, and the gauge field configurations used. I shall refer to the collaborations as FNAL [1, 2], KEK [3], NRQCD [4, 5], UKQCD [6, 7].

The FNAL group [1, 2] use improved heavy Wilson fermions for both $c\bar{c}$ and $b\bar{b}$ spectra on quenched gauge field configurations at several values of β , but principally using 100 configurations at $\beta = 6.1$.

The KEK [3] group use Wilson fermions for the $c\bar{c}$ spectrum but, for the first time, on gauge field configurations with 2 flavours of dynamical quarks. They have 75 configurations at $\beta = 5.7$.

The NRQCD group uses Non-relativistic QCD (NRQCD) to study $b\bar{b}$ [5] (with 100 quenched gauge field configurations at $\beta = 6.0$ from the Staggered Collaboration) and $c\bar{c}$ spectra [4] (with 100 quenched gauge field configurations at $\beta = 5.7$ from the UKQCD collaboration). The NRQCD action is improved at next-to-leading order.

The UKQCD collaboration has results for both improved heavy Wilson [7] ($c\bar{c}$ and extrapolating to $b\bar{b}$) and leading order NRQCD (for $b\bar{b}$) [6] principally at $\beta = 6.2$ on 60 quenched configurations.

The most complete spectrum is presented by the NRQCD collaboration, with both spin-averaged splittings and s and p hyperfine split-

tings for $b\bar{b}$ and $c\bar{c}$. I will quote results from other groups where available. The KEK group give only the 1S-1P spin-averaged splitting which is useful for the determination of a^{-1} and α_s . Their result provides a confirmation of the unquenching corrections that other groups employ to convert their quenched α_s to a physical one. This is discussed by El-Khadra [2] and I will not refer to the KEK results further.

1.1. NRQCD

NRQCD makes use of an expansion of the quark action in powers of v^2/c^2 where v/c is a typical velocity inside the heavy meson. $v^2/c^2 \sim 0.1$ for Υ and 0.3 for J/Ψ . The terms in the action can be ordered by power-counting rules [8].

The lowest order terms are simply

$$S_{lo} = \psi^\dagger D_t \psi + \psi^\dagger \frac{D^2}{2M} \psi \quad (1)$$

ψ is a 2-component quark field and the separate antiquark has the same action. The presence of a single time derivative means that the propagator can be calculated in one sweep through the lattice using a suitable time evolution equation [8] derived from a lattice version of the action.

This leading order action can already give useful results for spin-averaged splittings, i.e. splittings in which states of a given L are averaged over, weighted by the value of $2J+1$. These splittings can be compared to those extracted from a free Schrödinger equation with the usual central heavy quark potential. The leading order action has errors of 10% for $b\bar{b}$ and 30% for $c\bar{c}$. The UKQCD collaboration work with this action [6].

The lowest order spin-dependent terms (those with an explicit σ are

$$S_\sigma = \psi^\dagger \frac{g}{2M} \sigma \cdot B \psi + \psi^\dagger \frac{g}{8M^2} \sigma \cdot (D \times E - E \times D) \psi \quad (2)$$

The first of these gives s state hyperfine splittings ($^3S_1 - ^1S_0$) and the second p state hyperfine splittings (between χ states). Inclusion of these terms gives the hyperfine splittings to 10% for $b\bar{b}$ and 30% for $c\bar{c}$.

The NRQCD group includes also the next-to-leading contributions to the spin-averaged splittings. These are given by

$$S_{nlo} = \psi^\dagger \frac{D^4}{8M^3} \psi - \psi^\dagger \frac{ig}{8M^2} (D \cdot E - E \cdot D) \psi \quad (3)$$

In addition there are $\mathcal{O}(a^2)$ corrections that appear at the same level and are included through terms containing D^4 and D_i^4 [5]. Now spin-averaged splittings have systematic errors at the 1% level for $b\bar{b}$ and at the 10% level for $c\bar{c}$ from terms neglected in the heavy quark action.

All the terms above have been written with their tree level coefficients. Potentially large radiative corrections to these coefficients can be absorbed by a transformation of the gauge fields :

$$U_\mu \rightarrow \frac{U_\mu}{u_0} \quad (4)$$

This effectively includes in each term tadpole corrections to all orders [8]. These are assumed to be the dominant corrections [9] so that any further radiative corrections can be ignored at the next-to-leading order level. Explicit calculations [10] for the coefficients of D^4 and D_i^4 confirm this.

1.2. Heavy Wilson

The approach of heavy Wilson fermions is very similar to that of NRQCD (in fact, eventually identical). It begins by adding a clover term to the usual Wilson action to correct for errors at $\mathcal{O}(a)$.

$$S_{clover} = ic \frac{\kappa}{2} \bar{\psi} F_{\mu\nu} \sigma_{\mu\nu} \psi \quad (5)$$

Propagators are calculated by the usual methods for Wilson fermions. Convergence for heavy quarks (low κ) is fast but it still requires $\mathcal{O}(10)$ passes through the lattice. The FNAL group give c the value calculated from the dominant tadpole effects described above [1]. This value is $u_0^3 = 1.4$ at $\beta = 6.1$. The UKQCD group [7] give c its tree level value of 1.0.

Compared to the nlo NRQCD approach above, the heavy Wilson action has terms missing at $\mathcal{O}(a^2)$ and at order D^4 . However, there is some remnant of the required D^4 term present so the error should be less than the 10 % or 30% of lo NRQCD.

1.3. Gauge Field Configurations

There are two sources of systematic error present in standard quenched gauge field configurations which must be taken into account when

assessing the results of the FNAL, NRQCD and UKQCD groups. Luckily the two effects tend to counteract each other to some extent.

The first is the presence of $\mathcal{O}(a^2)$ errors in the usual Wilson plaquette gauge action. This can be corrected by the use of 2×1 or 2×2 plaquettes with an appropriate tadpole-improved coefficient.

We can estimate the size of these effects from a Schrödinger equation on the lattice using a lattice heavy quark potential (such as that from Bali and Schilling [11]). The $\mathcal{O}(a^2)$ errors in the gluon propagator show up as a lack of rotational invariance in the potential at small R . If we correct the $[4\sin^2(ka/2)]^{-1}$ of the one-gluon exchange term to $[4\sin^2(ka/2) + 4/3\sin^4(ka/2)]^{-1}$ and compare the spectra at fixed quark mass, we can obtain an estimate of the size of these $\mathcal{O}(a^2)$ errors. We find a 2% reduction of the spin-averaged $1P - 1S$ splitting when the corrections are included.

Another, more serious, source of error is that of the quenching of the gauge fields (at least partly removed in the KEK results). We are studying states mainly well below threshold for decay to heavy-light channels, so the principal effect of quenching is from the change in the shape of the heavy quark potential at small distances. The quenched coupling constant runs too fast to zero, so the potential close to $R = 0$ is not deep enough. Quenching then has the effect of raising states which are concentrated at short distances, such as s states. A comparison of results from a lattice Schrödinger equation with quenched and unquenched potentials from the MTc collaboration [14] gave a shift to the ratio of $2S - 1S$ to $1P - 1S$ of 10% (downwards on unquenching). The potentials did not have the same lattice spacing so an absolute shift for $1p - 1S$ could not be derived. We fixed the quark mass in physical units to 4.7 GeV in both cases.

In perturbation theory the s state hyperfine splitting is proportional to the square of the wavefunction at the origin and $\alpha_s(M)$. In the above analysis we did not find a big effect from quenching on the wavefunction at the origin. This disagrees with earlier results from El-Khadra [12] who used a continuum Richardson potential. We believe that the lattice Schrödinger approach may more accurately reflect what we will find in lat-

tice simulations. The effect of quenching on $\alpha_s(M)$ is to reduce it by 20%. Thus the lattice Schrödinger approach predicts an s state hyperfine splitting 20% too low in the quenched approximation and the continuum Richardson gives a 40% reduction.

2. RESULTS

There are two parameters to be fixed in all these calculations.

- a^{-1} : Fix from the spin-averaged spectrum which is relatively insensitive to M . (Compare the s and p splittings for $b\bar{b}$ and $c\bar{c}$ from the Particle Data Book [13]).
- M : Fix from the kinetic mass, M_2 , of the Υ or $J/\psi/\eta_c$ as appropriate. M_2 is given by the non-relativistic dispersion relation for the energy of a meson at finite momentum:

$$E(p) = M_1 + \frac{p^2}{2M_2} + \dots \quad (6)$$

2.1. $b\bar{b}$ results

Figure 1 shows the ‘spin-averaged’ spectrum for $b\bar{b}$ obtained by the NRQCD collaboration at $\beta = 6.0$ and the UKQCD collaboration (using leading order NRQCD) at $\beta = 6.2$. Since no experimental mass is available for the η_b (1S, or its radial excitations, 2S and 3S), no spin-averaging of s states is done by NRQCD. The experimental results, given as bars, are for the $\Upsilon(^3S_1)$, 1S, 2S and 3S. The UKQCD results are inevitably spin-averages since they have no spin-dependent terms in the action. For the p states a spin-average over the $^3P_{0,1,2}$ states, all known experimentally at 1P and 2P, is done. In addition the NRQCD collaboration have a mass for the as yet unseen 1P_1 state, the h_b .

By performing a bootstrap fit to the whole spectrum of Figure 1, the NRQCD collaboration obtain a value for the inverse lattice spacing, $a^{-1} = 2.4(1)$ GeV. The Q value for the fit is good at 0.4. Taking the ratio of plaquette values at $\beta = 6.0$ and $\beta = 6.2$ and using lattice perturbation theory gives a value for the inverse lattice spacing at $\beta = 6.2$ of 3.2(2) GeV. It is this value of

Figure 1. The spectrum for $b\bar{b}$ states plotted relative to the mass of the Υ (1S). Solid horizontal lines mark the experimental data for the 3S_1 and for the spin-average of the $^3P_{0,1,2}$ (χ_b) ground states and radial excitations. Grey horizontal lines mark the expected masses of the 1P_1 (h_b) states. The vertical scale is in GeV. Squares mark the results from the NRQCD collaboration. The stars are those from the UKQCD collaboration (using leading order NRQCD). Their results are all for spin-averaged masses. The cross on $\Upsilon(1S)$ shows that its mass was assumed in the results.

a^{-1} that has been used to convert the UKQCD results from lattice to physical units. The FNAL group have calculated so far a value only for the 1P-1S splitting for $b\bar{b}$ and they use this to determine a^{-1} , giving no independent predictions to appear in Figure 1. They obtain 2.7(2)GeV at $\beta = 6.1$, a value in agreement (using perturbative extrapolations) with the NRQCD value above.

Notice that all these values of a^{-1} are larger than those quoted at similar β values from the string tension or light hadron spectra. This simply reflects the scale dependence of the determination of a^{-1} from the quenched approximation, when the long distance heavy quark potential is too steep and the short distance potential not steep enough. The difference is that the short

distance potential can be corrected perturbatively for its quenching errors.

Figure 1 shows a good fit to experimental data [13] within the errors. The statistical and systematic errors of the NRQCD calculation [5] are such that it is almost possible to see the effect of the quenched approximation. The 1P level is slightly too low and the 2S slightly too high. Estimates from a lattice Schrödinger equation described above give a correction for quenching and gluonic $\mathcal{O}(a^2)$ effects that produces a very good fit to experiment.

Figure 2 shows the hyperfine spectrum from the NRQCD collaboration for $b\bar{b}$ 1P states compared to experiment. Separate results are given for different components of the 3P_2 and 3P_1 states. The fit is good, and provides a stringent test of the coefficient of the $\sigma.(D \times E)$ term in the spin-dependent action. Using tree-level coefficients without correction for tadpoles would give splittings reduced by a factor of 50%. Values for the (unobserved) 1S hyperfine splitting are 29(4) MeV for NRQCD, 25(4) MeV for FNAL and 18(1) MeV for UKQCD using heavy Wilson with a tree-level improvement coefficient. It seems likely that the UKQCD value is too low.

A pole mass can be extracted for the b quark from a perturbative analysis of the dispersion relation for the Υ at finite momentum. A fit to the energy of propagators at zero and non-zero p gives the Υ mass in lattice units and its zero-momentum energy (M_1 and M_2 in equation 7). However, M_1 and M_2 should be related by

$$M_2 = 2(Z_M M_b^0 - E_0) + M_1 \quad (7)$$

where Z_M and E_0 are perturbatively calculable mass renormalisation and energy shift parameters and M_b^0 is the bare heavy quark mass. The NRQCD collaboration [5] compares the results for M_2 from equation 8 and the value extracted directly using equation 7. They agree to 1% for a variety of bare heavy quark masses. This agreement is at the level expected for a *nlo* calculation and should be seen as a triumph for the whole method. It requires both that the *nlo* coefficients be correct and that the lattice perturbation theory is well-behaved. Thus the assumption of tadpole dominance of the coefficients is correct and

Figure 2. The hyperfine spectrum of the $b\bar{b}$ 1P states, relative to the spin-average of the $^3P_{0,1,2}$ (χ_b) states. The horizontal lines mark experimental data; the vertical scale is in MeV. The squares are results from the NRQCD collaboration, with error bars where these would be visible.

there are no unexpectedly large perturbative or non-perturbative contributions to Z_M or E_0 . The numbers used for Z_M and E_0 have been calculated to $\mathcal{O}(g^2)$ for the action used by the NRQCD collaboration [10] and using the scheme and scale for g^2 advocated by Mackenzie and Lepage [9].

Fixing M_b^0 so that the experimental value for M_Υ is obtained then yields a value for the renormalised or pole b mass, $Z_M M_b^0$. This value is 4.7(1) GeV where the main source of error is that of higher order corrections to E_0 . Simulation errors are not significant.

2.2. $c\bar{c}$ results

New results for the $c\bar{c}$ spectrum were presented only by the NRQCD collaboration. Their results are shown in Figure 1 of the talk by A.J. Lidsey [4]. Now experimental results are available for both the $\eta_c(^1S_0)$ and the $h_c(^1P_1)$. The spin-averaged spectrum is therefore presented with the spin-averaged s state at zero. There are two s states below threshold for decay to heavy-light channels. The 2S state is fit using a 3-exponential

fit to 2 correlation functions. The fit to experiment is good and certainly within systematic errors of 10%. An estimate for the position of a d state was obtained by calculating a correlation function for the 1D_2 . This is compared to the posited 3D_1 state, $\psi(3770)$. The 1D_2 is higher, as expected. There may also be significant coupling to decay channels here which would tend to distort the spectrum in the quenched approximation.

Figure 3 shows the $c\bar{c}$ hyperfine spectrum. Both s and p states are given relative to their respective spin-averages. For the s state hyperfine splitting there is some disagreement between groups. The NRQCD collaboration obtains a value of 96(4)MeV, the FNAL collaboration, 64(4)MeV and the UKQCD collaboration, 50(1)MeV. The experimental value is 116MeV. There is a 30% systematic error in the calculations from higher order terms in the quark action and also a systematic effect from quenching which tends to reduce the value. If the effect of quenching is a 40% reduction then the NRQCD value looks high. If a 20% effect then the other values look low. The NRQCD result has the smallest systematic error from the quark action, which should give the best accuracy in determining the quark mass (something the hyperfine splitting is sensitive to). On the other hand the FNAL collaboration have results for the hyperfine splitting for different values of β and see no strong systematic effect from $\mathcal{O}(a^2)$ terms that they have not included. This disagreement needs to be resolved.

3. CONCLUSIONS

Heavy quark spectroscopy can provide a stringent test of QCD. The $b\bar{b}$ spectrum can be calculated to the 1% level using an action correct to nlo in v^2/c^2 . Such an action can be provided within an NRQCD or heavy Wilson approach. Current results in NRQCD are at this level in the quark action. There are remaining systematic errors present from the gluon configurations used. These should be corrected in future calculations.

Acknowledgements

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Figure 3. The hyperfine spectrum of the $c\bar{c}$ 1S and 1P (χ_c) states, relative to their respective spin-averages. The horizontal lines mark experimental data; the vertical scale is in MeV. The squares are results from the NRQCD collaboration, the circles from the FNAL collaboration and stars from the UKQCD collaboration (heavy Wilson). Error bars are shown where they would be visible.

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